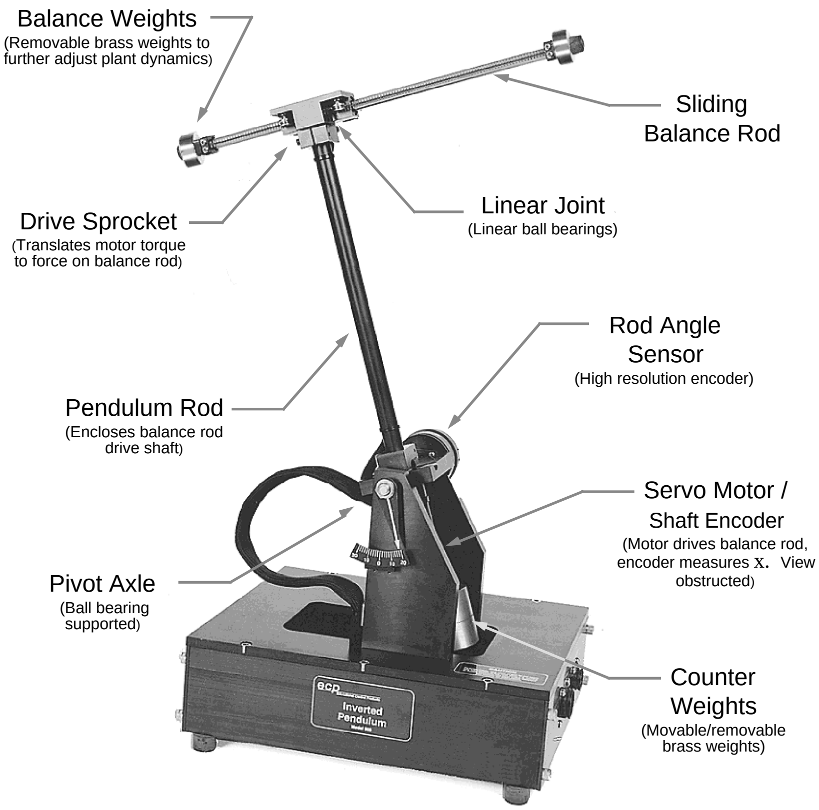
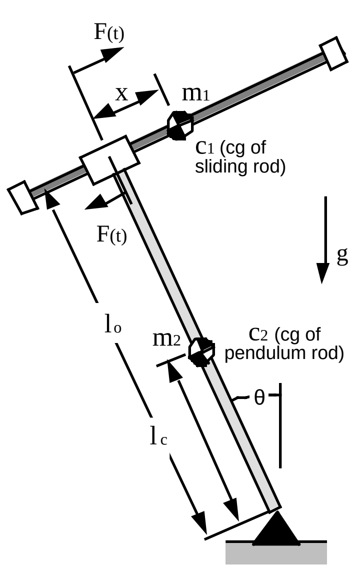
Model 505
ECP Inverted Pendulum



This unique ECP design vividly demonstrates the need for and effectiveness of closed loop control. It is *not* the conventional rod-on-cart inverted pendulum, but rather, it steers a horizontal balancing rod in the presence of gravity to control the vertical pendulum rod. As detailed in the manual, the plant has both right half plane poles and zeros as well as kinematic and gravitationally coupled nonlinearities. By adjusting mass properties, these roots may be varied to make the control problem range from being relatively simple to theoretically impossible! The system includes removable and adjustable moment arm counter-weights on the vertical and horizontal rods for quick adjustment of the plant dynamics. It features linear and rotary ball bearings at the joints for low friction and consistent dynamic properties.

Dynamics: 4th order, nonminimum phase, open loop unstable, kinematic & gravitationally coupled nonlinearities
Parameter Adjustment: Adjustable vertical and horizontal rod mass, inertia, and CG offset.
I/O: SISO, SIMO,
Poles: Adjustable 0.4-1.2 Hz
Feedback: High res. encoders (16,000 count/rev, θ, 44,000 count/m, x)
Actuator: High Torque density, rare earth magnet type
Bench-top size: 30x30x40 cm. (12x12x16 in.)
Safety Features: Travel limit microswitches (horizontal rod) , fail-safe shutdown, limit cushions (vertical rod), amplifier over-current protection. In firmware (complete system only): i^2t thermal protection

Plant Model	***Dynamic Equations***		***Characteristics***
	***"Exact"***	$m_1\ddot{x}(t)+m_1 l_o\ddot{\theta}(t) - m_1 x(t)\dot{\theta}(t)^2 - m_1 g\sin\theta(t) = F(t)$ $m_1 l_o\ddot{x}(t)+J_o(x)\ddot{\theta}(t)+2m_1 x(t)\dot{x}(t)\dot{\theta}(t) - (m_1 l_0+m_2 l_c)g\sin\theta(t) - m_1 g x(t)\cos\theta(t) = 0$ $J_o(x) = J_1+m_1(l_o^2+x^2)+J_1+m_2 l_c^2$	• Nonlinearities in kinematic constraints and coordinate dependent mass properties.
	Linearized Time Domain	$m_1\ddot{x}(t)+m_1 l_o\ddot{\theta}(t) - mg\,\theta(t) = F(t)$ $m_1 l_o\ddot{x}(t)+J_o^*\ddot{\theta}(t) - (m_1 l_c+m_2 l_o)g\theta(t) - m_2 g x(t) = 0$ $J_o^* = J_o\big\vert_{x=0}$	• Linearization about x = 0, θ = 0 shown to be valid for many control schemes.
	S-Domain	$\frac{\theta(s)}{F(t)} = \frac{-(l_o s^2 - g)}{(J_o^* - m_1 l_o^2)s^4 + (m_2 l_o - m_1 l_c)g s^2 - m_2 g^2}$	• One RHP, 2 oscillatory poles. • Nonmin phase (RHP zero). • Attainable bandwidth bounded from above and below by RHP roots.